

**MATH 464 (THEORY OF PROBABILITY)**  
**HOMEWORK 3**

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- (1) In a group of students, 25% smoke cigarettes, 60% drink alcohol, and 15% do both. What fraction of students have at least one of these bad habits?
- (2) Three couples that were invited to dinner will independently show up with probabilities 0.9, 0.8, and 0.75. Let  $N$  be the number of couples that show up. Calculate the probability that  $N = 3$  and that of  $N = 2$ .
- (3) The probability that a married man votes is 0.45, the probability that a married woman votes is 0.4, and the probability a woman votes given that her husband votes is 0.6. What is the probability that (a) both vote, (b) a man votes given that his wife votes?
- (4) Show that if  $A$  and  $B$  are independent events, then  $A$  and  $B^c$  are independent,  $A^c$  and  $B$  are independent, and  $A^c$  and  $B^c$  are independent.
- (5) If 5% of men and 0.25% of women are color blind, what is the probability that a randomly selected person is color blind?
- (6) Statistics show that 3% of men smoke but only 1% of women do. During a non-smoking flight, one passenger is smoking in the restroom. There are 40 male and 60 female passengers on the plane. What is the chance that the person smoking in the restroom is a man?
- (7) On a multiple-choice exam with four choices for each question, a student either knows the answer to a question or marks it at random. Suppose the student knows answers to 60% of the exam questions. If she marks the answer to question 1 correctly, what is the probability that she knows the answer to that question?
- (8) Suppose that the number of children in a family is 1, 2, or 3, with probability  $1/3$  each, and boys and girls appear equally likely. Little Bobby has no brothers. What is the probability that he is an only child?
- (9) Three factories,  $F_1$ ,  $F_2$ , and  $F_3$  produce computer chips.  $F_1$  produces 50% of all the chips in the market,  $F_2$  and  $F_3$  produce 40% and 10% respectively. 1% of the chips made by  $F_1$  are defective. For  $F_2$  and  $F_3$ , the rate of defective chips are 2% and 3%, respectively. Suppose Bob's computer has a defective chip. Which factory most likely to have produced it?
- (10) In a certain city 30% of the people are Conservatives, 50% are Liberals, and 20% are Independents. In a given election,  $2/3$  of the Conservatives voted, 80% of the Liberals voted, and 50% of the Independents voted. If we pick a voter at random, what is the probability he/she is Liberal?
- (11) Let  $\Omega$  be a sample space, and  $\mathcal{F}$  be the corresponding event space ( $\sigma$ -field on  $\Omega$ ). Suppose that  $\mathbb{P}$  is a mapping from  $\mathcal{F}$  into  $[0, 1]$  satisfying

- (i)  $\mathbb{P}(\Omega) = 1, \mathbb{P}(\emptyset) = 0,$
- (ii) if  $A, B \in \mathcal{F}$  and  $A \cap B = \emptyset$  then  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B),$
- (iii) if  $A_1, A_2, \dots \in \mathcal{F}$  and  $A_i \subseteq A_{i+1}$  for  $i = 1, 2, \dots$  then

$$\mathbb{P}(A) = \lim_{i \rightarrow \infty} \mathbb{P}(A_i), \text{ where } A = \bigcup_{i=1}^{\infty} A_i.$$

Prove that  $\mathbb{P}$  is a probability measure on  $(\Omega, \mathcal{F})$ .